Topic 13: Writing and Solving Inequalities

You just explored what it means for two quantities to be equal. When you write an equation you are comparing the value of two equal quantities. Sometimes you need to compare two quantities that are not equal.

An **inequality** is a mathematical sentence that compares the values of two expressions that are not equal. Instead of using an equal sign, you use an inequality symbol.

Inequality Symbols

$<$	less than
$>$	greater than
$\leq$	less than or equal to
$\geq$	greater than or equal to
$\neq$	not equal to

Problem 13.1 Writing Inequalities

A. For each situation, first define a variable. Then represent the situation with an inequality.

1. The height of a child must be at least 48 inches to ride a roller coaster.

2. The speed limit on the road is less than or equal to 45 miles per hour.

3. A piece of luggage must be less than 60 pounds.

4. You must be at least 13 years old to see a movie rated PG-13.

Exercises

For each situation, define a variable. Then write an inequality to model each situation.

1. Ana has a car. She wants to limit herself to driving at most 500 miles per month.

2. The Simon family's car emits 0.75 pounds of CO_2 per mile. It emits 2 pounds of CO_2 when it is started. The Simons want to limit their emissions to at most 100 pounds of CO_2 per use of the car.

3. An online discount costs $50 per month. It decreases $2 for every person you recommend to sign-up. You want to keep the total cost below $35.

You know that to maintain an equality, you can add, subtract, multiply or divide both sides of the equality by the same number.

Problem 13.2 Properties of Inequalities

A. Consider the inequality $6 < 20$.

   1. Does the inequality remain true when you add or subtract each side by the same number?

   2. Does it remain true when you multiply or divide by a positive number? A negative number?

   3. What seems to be true about properties of inequalities?

B. Sally did the following work to solve the inequality $-4x + 2 < 10$.

$$-4x + 2 < 10$$

$-4x + 2 - 2 < 10 - 2$	Subtract 2 from each side.
$-4x < 8$	Simplify each side.
$\frac{-4x}{-4} > \frac{8}{-4}$	Divide each side by -4. Reverse the inequality symbol.
$x > -2$	Simplify.

Is she correct? Explain.

Problem 13.3 Solving Inequalities

Each month the expenses of Fabulous Fabian's Bakery from Problem 3.5 in *Moving Straight Ahead* must be less than a set amount. Recall that the expenses E to make n cakes in a month is represented by $E = 825 + 3.25n$.

A. Fabian needs to keep expenses this month less than $2,200.

   1. Write an inequality using this information.

   2. Solve your inequality. What does this mean for Fabian's bakery?

B. Check to make sure your answer is reasonable.

Exercises

Solve each inequality.

   1. $3x + 17 < 47$ **2.** $14x - 23 < 5x + 13$

   3. $43 < 8t - 9$ **4.** $182 < -4m + 2$

   5. $-6c + 9 < 25$ **6.** $3,985 + 59 < 995 + 14.95d$

   7. Vince finds out that his family's SUV emits an average of 1.25 pounds of CO_2 per mile. Suppose Vince's family wants to limit CO_2 to at most 600 pounds per month.

     a. Write an inequality using this information.

     b. Solve your inequality. What does the solution mean for the Vince's family?

     c. Check to make sure your answer is reasonable.

Topic 13: Writing and Solving Inequalities

Mathematical Goals

- Write inequalities to represent situations
- Solve inequalities in one variable

Teaching Guide

Students should have a clear understanding of equations before they start working with inequalities, as many of the same rules apply. While students should be familiar with the similarities between equations and inequalities, it is also crucial that they recognize the differences. Remind students throughout the lesson to use the correct inequality symbols since some students may use equal signs out of habit.

Many students will also need to practice writing verbal statements as algebraic expressions. Sufficient time should be spent familiarizing students with the meanings of the inequality symbols. One possible classroom activity is to divide the students into pairs and instruct them to take turns writing situations for their partner to translate into algebraic expressions.

When solving inequalities, many students struggle with solving by multiplication and division. You may choose to do several examples as a class to highlight the importance of reversing the inequality sign when multiplying or dividing by a negative number.

After Problem 13.1, ask:

- *What are some common phrases that are used to describe "greater than or equal to"? To describe "less than or equal to"?*
- *How would you rewrite an inequality with the variable on the opposite side of the inequality symbol?*

Explain to students what it means for a number to satisfy an inequality, or be a solution to the inequality. Have students find several different solutions to the same inequality. Point out to students the difference between the solutions for pairs of inequalities like $x < 2$ and $x \le 2$. Then have students discuss what the solutions of the inequality mean in a real-world situation.

Summarize the example before Problem 13.2 by asking:

- *What values could you use to check your solution to the inequality?*

During Problem 13.2, ask:

- *What information does the number 825 represent?*
- *What information does the number 3.25 represent?*
- *For which variable in the inequality should you substitute 2,200?*

Homework Check

When reviewing Exercise 6, ask:

- *What was your first step in solving this inequality?*
- *Could you still get the correct answer without doing this first?*

Assignment Guide for Topic 13

Core Problem 13.1 Exercises 1–3, Problem
13.2–13.3 Exercises 1–6
Advanced Problem 13.2–13.3 Exercise 7

Answers to Topic 13

Problem 13.1

A. 1. Let h = the child's height in inches; $h \geq 48$.

  2. Let s = speed in miles per hour; $s \leq 45$.

  3. Let w = the weight of the piece of luggage in pounds; $w < 60$.

  4. Let a = your age in years; $a \geq 13$.

Exercises

  1. Let m = the number of miles Ana drives; $m \leq 500$

  2. Let m = the number of miles the Simons drive; $2 + 0.75m \leq 100$

  3. Let p = the number of people you recommend; $50 - 2p < 35$

Problem 13.2

A. 1. yes

  2. yes; no

  3. Inequalities remain true when you add or subtract each side by the same number, and when you multiply or divide by a positive number.

B. Yes; Sally reversed the inequality symbol when she divided each side by a negative number.

Problem 13.3

A. 1. $2{,}200 > 825 + 3.25n$

  2. $n < 423.08$; the bakery must make fewer than 423 cakes this month.

B. Check students' work.

Exercises

  1. $x < 10$

  2. $x < 4$

  3. $t > 6.5$

  4. $m < -45$

  5. $c > -2\frac{2}{3}$

  6. $d > 203.95$

  7. a. $1.25m \leq 600$

  b. $m \leq 480$; Vince's family must drive no more than 480 miles per month.

  c. Check students' work; students should show that they checked at least one value that makes the inequality true, as well as one that makes the inequality false.